■ Easy steps to complete the square

Rewrite the function $f(x) = a\,x^2 + b\,x + c$ in the form $f(x) = a(x + h)^2 + k$

Step 1: group the terms that contain the variable

$$f(x) = (a\,x^2 + b\,x) + c \tag{1}$$

Step 2: if $a \neq 1$, then factor it out

$$f(x) = a(\,x^2 + \frac{b}{a}\,x) + c \tag{2}$$

Step 3: add $(\frac{b}{2\,a})^2$ inside the parenthesis and subtract $\frac{b^2}{4\,a}$ outside the parenthesis

$$f(x) = a\Big(\,x^2 + \frac{b}{a}\,x + (\frac{b}{2\,a})^2\Big) + c - \frac{b^2}{4\,a} \tag{3}$$

Step 4: rewrite $x^2 + \frac{b}{a}\,x + (\frac{b}{2\,a})^2$ as $(x + \frac{b}{2\,a})^2$

$$f(x) = a(x + \frac{b}{2\,a})^2 + c - \frac{b^2}{4\,a} \tag{4}$$

Step 5: do the arithmetic required to simplify $c - \frac{b^2}{4\,a}$ to a single number

$$f(x) = a(x + \frac{b}{2\,a})^2 + k \tag{5}$$

■ **Example:** $F(x) = 5\,x^2 + 3\,x + 8$

Step 1: group the terms that contain the variable

$$f(x) = (5\,x^2 + 3\,x) + 8 \tag{6}$$

Step 2: if $a \neq 1$, then factor it out

$$f(x) = 5\,(x^2 + \tfrac{3}{5}\,x) + 8 \tag{7}$$

Step 3: add $(\tfrac{3}{10})^2$ inside the parenthesis and subtract $\tfrac{9}{20}$ outside the parenthesis

$$f(x) = 5\,(x^2 + \tfrac{3}{5}\,x + \tfrac{9}{100}) + 8 - \tfrac{9}{20} \tag{8}$$

Step 4: rewrite $x^2 + \tfrac{3}{5}\,x + \tfrac{9}{100}$ as $(x + \tfrac{3}{10})^2$

$$f(x) = 5\,(x + \tfrac{3}{10})^2 + 8 - \tfrac{9}{20} \tag{9}$$

Step 5: do the arithmetic required to simplify $8 - \tfrac{9}{20}$ to a single number

$$f(x) = 5\,(x + \tfrac{3}{10})^2 + \tfrac{151}{20} \tag{10}$$